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Research Article

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Prey and Predator Model
A Study of the Transition from the Deterministic Model to the Stochastic Counterpart

Asmahan M. A. Billiwa [1*] Abdassalam B. H. Aldaikh[1]
[1]Department of Mathematics, Faculty of Science, University of Omar Al-Mukhtar, El-Beida-Libya

***Correspondence author:** *aldaikh.1962@omu.edu.ly*

ABSTRACT

In this paper, a deterministic model was proposed and analyzed that describes the interaction between two objects representing the prey and the predator. The focus was on studying the stability of this model by determining the states that the system could be in. In addition, numerical simulations were used to clarify the behavior of the system. Converting the deterministic model to a stochastic model, is a main goal of this study, to explore and analysis the random interactions between prey and predator. The aim of this transformation was to understand the more complex dynamics that may appear in real environmental systems. Numerical simulations by MATLAB tools, were also used to illustrate the solutions behavior of the system both in its deterministic and stochastic situations.

***Keywords**: Prey and predator model, stochastic model, stability, covariance matrix, diffusion matrix.*

نموذج الفريسة والمفترس
دراسة التحول من النموذج الحتمي إلى النموذج العشوائي

أسمهان محمد بلليوا[1] وعبدالسلام بوحويش[1*]
[1]قسم الرياضيات، كلية العلوم، جامعة عمر المختار، البيضاء، ليبيا
***للمراسلة** aldaikh.1962@omu.edu.ly

الملخص

في هذا البحث تم اقتراح وتحليل نموذج حتمي يصف التفاعل بين كائنين يمثلان الفريسة والمفترس. وتم التركيز على دراسة استقرار هذا النموذج من خلال تحديد الحالات التي يمكن أن يكون عليها النظام. بالإضافة إلى ذلك، تم استخدام المحاكاة العددية لتوضيح سلوك النظام. ان الهدف الرئيسي من هذه الدراسة هو تحويل النموذج الحتمي إلى نموذج عشوائي، وذلك لتحليل التفاعلات العشوائية بين الفريسة والمفترس، مما يؤدي الى فهم الديناميكيات الأكثر تعقيدا التي قد تظهر في النظم البيئية الحقيقية. كما تم استخدام المحاكاة العددية عن طريق أدوات برنامج الماتلاب، لتوضيح سلوك حلول هذا النظام في حالتيه المحددة والعشوائية.

الكلمات المفتاحية: نموذج الفريسة والمفترس، النموذج العشوائي، استقرار، مصفوفة التغاير، مصفوفة الانتشار.

INTRODUCTION

Predation is an ecological process in which two organisms the predator and the prey interact with one another. Individuals of one species consuming living members of another species is known as predation. Species that rely on other creatures, or prey, for development, nutrition, and reproduction are known as predators (Gese & Knowlton, 2001). They are often active, bigger than their victims, and require many preys to complete their life cycle (Seni & Halder, 2022). Many insects, spider, mollusks, fish, amphibian, reptilian, avian, and mammal species are examples of imported invasive predators that are found across the animal kingdom (Mordukhai, 1969).

One of the most misinterpreted ecological processes is predation. Predation and impact are often confused, with Predators significantly affect their ecosystems as a whole, and with most people thinking that killing a single animal will always have a detrimental effect on the population. This perspective is frequently incorrect and fails to take into account the intricacy of predation at the individual, population, and social levels (Bender, 2018).

Predators are becoming more and more acknowledged globally for their role in ecosystem function. In addition to having a significant influence on their prey, predators may also have a cascade effect on several species and ecological processes. Although they may have saved many species, predators have played a significant role in the decline and extinction of countless others (Glen & Dickman, 2014), and non-native predators can have an impact on ecosystems' trophic dynamics and biodiversity (Sanders & Mills, 2022), hence, these days, preserving the ecological balance of nature is a global priority.

There is debate on the part predation plays in the dynamics of prey groups. A general lack of information about most ecosystems and other environmental variables hamper our understanding of predator-prey dynamics. Predation is not the only factor that may control or restrict prey numbers; other variables can determine how much predation impacts prey populations. Moreover, certain elements could create invisible generational effects or even time delays. Predation's involvement in animal population dynamics and some of the elements influencing predator-prey interactions are reviewed here. We also try to identify areas of current professional discussion and potential future directions for improving our knowledge of predator-prey interactions. To forecast the negative effects of managing endangered species, complex, data-hungry ecological models are frequently employed. Regretfully, this method is not feasible in many systems that lack the data necessary to calibrate, verify, or accurately define ecosystem interactions. We developed an alternative method that uses a straightforward, practical model to inform choices in systems with limited resources and data (Maiti & Samanta, 2016).

The modeling of herd behavior in populations has gained more attention in recent years. The majority of research focuses on predator-prey models. The prey-predator paradigm, in which both the predator and the prey display herd behavior, has been examined in this work. The study of deterministic actions is conducted. Gaussian white noise is then added to examine the impact of the fluctuating environment.

METHODOLOGY

The mathematical model

Let $x(t)$ represent the prey's density and $y(t)$ represent the predator's density. It is assumed that both predators and prey reside in herd. These factors encourage us to present the following predator-prey system using the following collection of nonlinear ordinary differential equations as a model (Plein et al, 2022 and Allen, 2010).

$$\begin{cases} \frac{dx}{dt} = f_1(x,y) = rx\left(1 - \frac{x}{K}\right) - \frac{xy}{x+c} \\ \frac{dy}{dt} = f_2(x,y) = y\left(-b + \frac{x}{x+c}\right) \end{cases}.$$

Where $\boldsymbol{r}, \boldsymbol{K}, \boldsymbol{c}$ and $\boldsymbol{b}$ are positive constants, $\boldsymbol{x}$ represents the density of the prey (for example, Sheep) and y represents the density of the predator (for example, Wolves), Table 1.

Table1. Description of state variables and parameters of the proposed model

Variable	Description
x	Prey density
y	Predator density.
r	Prey growth rate
k	Carrying capacity
c	Available resources
b	Mortality rate of predators

The dynamics underlying this model is the following:

1- There will be an increase in prey in the absence of a predator (logistic growth).

$$\frac{dx}{dt} = \left(r - \frac{rx}{K}\right)x.$$

2- The number of wolves declines with the lack of prey until it eventually goes extinct.

$$\frac{dy}{dt} = -by\,.$$

3- As a result of the population densities, the number of meetings between the two species increases the population of hunters and decreases the population of prey.

Before creating the stochastic model for the system, we will study the stability of this system at certain points (Chou & Friedman, 2016).

1- The equilibrium points:

$$\frac{dx}{dt} = 0 \Rightarrow rx\left(1 - \frac{x}{K}\right) - \frac{xy}{x+c} = 0 \Rightarrow rx - \frac{rx^2}{K} - \frac{xy}{x+c} = 0 \Rightarrow x\left(r - \frac{rx}{K} - \frac{y}{x+c}\right) = 0.$$

$$\Rightarrow x = 0 \; or \; \left(r - \frac{rx}{K} - \frac{y}{x+c}\right) = 0 \Rightarrow r - \frac{rx}{K} - \frac{y}{x+c} = 0 \qquad (1)$$

$$\frac{dy}{dt} = 0 \Rightarrow y\left(-b + \frac{x}{x+c}\right) = 0 \Rightarrow y = 0 \; or \; \left(-b + \frac{x}{x+c}\right) = 0 \Rightarrow -b + \frac{x}{x+c} = 0 \qquad (2).$$

From (2) $x^* = \frac{cb}{(1-b)}$ and from (1) $y^* = rcm$ such that $m = \frac{(K-bc-Kb)}{K(1-b)^2}$.

Hence the equilibrium points are: (0,0), (k ,0), (x^*, y^*),

2- The Jacobian matrix:

$$J = \begin{pmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} \end{pmatrix} = \begin{pmatrix} \mathrm{r} - \frac{2\mathrm{rx}}{\mathrm{K}} - \frac{\mathrm{cy}}{(\mathrm{x+c})^2} & -\frac{\mathrm{x}}{\mathrm{x+c}} \\ \frac{\mathrm{cy}}{(\mathrm{x+c})^2} & -\mathrm{b} + \frac{\mathrm{x}}{\mathrm{x+c}} \end{pmatrix}.$$

(i) $J(0,0)\; is \begin{pmatrix} r & 0 \\ 0 & -b \end{pmatrix}$.

(ii) $J(k\,,0)\ is\ \begin{pmatrix} -r & -\frac{k}{k+c} \\ 0 & -b+\frac{k}{k+c} \end{pmatrix}$.

(iii) $J(\,x^{*},y^{*})\ is\ \begin{pmatrix} rb(\frac{(K-kb-bc-c)}{K(1-b)}) & -b \\ \frac{r}{K}(K-bc-Kb) & 0 \end{pmatrix}$.

3- Stability study:

(i) at (0,0) then

$$|J-\lambda I|=0.$$

$$\left|\begin{pmatrix} r & 0 \\ 0 & -b \end{pmatrix}-\lambda\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right|=0.$$

$$\left|\begin{pmatrix} r & 0 \\ 0 & -b \end{pmatrix}-\begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}\right|=0.$$

$$\begin{vmatrix} r-\lambda & 0 \\ 0 & -b-\lambda \end{vmatrix}=0.$$

$$(r-\lambda)(-b-\lambda)-0=0.$$

$$(r-\lambda)=0\ or\ (-b-\lambda)=0.$$

$$\lambda_1=r>0.$$

$$\lambda_2=-b<0.$$

Then the system at (0,0) is saddle, Table 2.

(ii) at (k ,0) then

$$|J-\lambda I|=0.$$

$$\left|\begin{pmatrix} -r & -\frac{k}{k+c} \\ 0 & -b+\frac{k}{k+c} \end{pmatrix}-\lambda\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right|=0.$$

$$\left|\begin{pmatrix} -r & -\frac{k}{k+c} \\ 0 & -b+\frac{k}{k+c} \end{pmatrix}-\begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}\right|=0.$$

$$\begin{vmatrix} -r-\lambda & -\frac{k}{k+c}-0 \\ 0 & (-b+\frac{k}{k+c})-\lambda \end{vmatrix}=0.$$

$$(-r-\lambda)\left((-b+\frac{k}{k+c})-\lambda\right)-0=0.$$

$$(-r-\lambda)=0\ or\ \left((-b+\frac{k}{k+c})-\lambda\right)=0.$$

$$\lambda_1=-r<0.$$

$$\lambda_2=(\frac{k}{k+c}-b).$$

hence the system at $(k\,,0)$ is

- Stable if $(\frac{k}{k+c} < b)$
- Saddle unstable if $\left(\frac{k}{k+c} > b\right)$.

(iii) at (x^*, y^*) then

$$|J - \lambda I| = 0.$$

$$\left|\begin{pmatrix} rb(\frac{(K-kb-bc-c)}{K(1-b)}) & -b \\ \frac{r}{K}(K-bc-Kb) & 0 \end{pmatrix} - \lambda\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right| = 0.$$

$$\left|\begin{pmatrix} rb(\frac{(K-kb-bc-c)}{K(1-b)}) & -b \\ \frac{r}{K}(K-bc-Kb) & 0 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}\right| = 0.$$

$$\begin{vmatrix} (rb(\frac{(K-kb-bc-c)}{K(1-b)})) - \lambda & -\mathrm{b} - 0 \\ (\frac{r}{K}(K-bc-Kb)) - 0 & 0 - \lambda \end{vmatrix} = 0.$$

$$\left((rb(\frac{(K-kb-bc-c)}{K(1-b)})) - \lambda\right)(-\lambda) - (-b)((\frac{r}{K}(K-bc-Kb))) = 0.$$

$$\left(-\left(rb\left(\frac{(K-kb-bc-c)}{K(1-b)}\right)\right)\lambda + \lambda^2\right) + (b)((\frac{r}{K}(K-bc-Kb))) = 0.$$

$$\left(\lambda^2 - \left(rb\left(\frac{(K-kb-bc-c)}{K(1-b)}\right)\right)\lambda\right) + (b)((\frac{r}{K}(K-bc-Kb))) = 0.$$

$$\lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

$$\lambda_{1,2} = \frac{-(-rb\left(\frac{(K-kb-bc-c)}{K(1-b)}\right)) \pm \sqrt{(-rb\left(\frac{(K-kb-bc-c)}{K(1-b)}\right))^2 - 4(1)(b)((\frac{r}{K}(K-bc-Kb))}}{2(1)}.$$

$$\lambda_{1,2} = \frac{(rb\left(\frac{(K-kb-bc-c)}{K(1-b)}\right)) \pm \sqrt{(rb\left(\frac{(K-kb-bc-c)}{K(1-b)}\right))^2 - ((\frac{4rb}{K}(K-bc-Kb))}}{2}.$$

$$\lambda_1 = \frac{\left(rb\left(\frac{(K-kb-bc-c)}{K(1-b)}\right)\right) + \sqrt{(rb\left(\frac{(K-kb-bc-c)}{K(1-b)}\right))^2 - ((\frac{4rb}{K}(K-bc-Kb))}}{2}.$$

$$\lambda_2 = \frac{\left(rb\left(\frac{(K-kb-bc-c)}{K(1-b)}\right)\right) - \sqrt{(rb\left(\frac{(K-kb-bc-c)}{K(1-b)}\right))^2 - ((\frac{4rb}{K}(K-bc-Kb))}}{2}.$$

Table 2. Stability Properties of Nonlinear System

The eigenvalues		Stability
Real	$\lambda_1 < \lambda_2 < 0$	Asymptotically stable
	$\lambda_1 = \lambda_2 < 0$	Asymptotically stable
	$\lambda_1 = \lambda_2 > 0$	Unstable
	$\lambda_1 > \lambda_2 > 0$	Unstable
	$\lambda_1 < 0 < \lambda_2$	Unstable (saddle)
Complex $\boldsymbol{\lambda_{1,2} = a \pm i\beta}$	$a < 0$	Stable spiral center
	$a > 0$	Unstable
	$a = 0$	Indeterminate

Prey numbers begin low and then sharply increase, indicating either a drop in predators or an increase in the availability of resources. This rise suggests favorable conditions that support the prey's development. A decrease in prey may be the cause of the fall in predator populations, as this has a detrimental effect on predators' access to food. As a result, it may be said that the system behaves unstablely since the quantities of predators and prey change on a regular basis, reflecting their constant dynamic interactions, Figure 1.

Now, we will examine how random elements may alter the system's behavior and how they may impact the potential outcomes.

The stochastic model for the proposed system:

Rewrite the system as

$$\begin{cases} \frac{dx}{dt} = rx - \left(\frac{rx^2}{K} + \frac{xy}{x+c}\right) \\ \frac{dy}{dt} = -by + \frac{xy}{x+c} \end{cases}.$$

1- Probabilities associated with changes in the predator – prey model are summarized in Table 3.

Table 3. Probabilities associated with changes in the predator – prey model

Changes, $\Delta \boldsymbol{x_i}$	Probability,$\boldsymbol{p_i}$
$(1,0)^{tr}$.	$\boldsymbol{rx\,\Delta t}$.
$(-1,0)^{tr}$.	$\left(\frac{rx^2}{K} + \frac{xy}{x+c}\right)\Delta t$.
$(0,-1)^{tr}$.	$\boldsymbol{by\,\Delta t}$.
$(0,1)^{tr}$.	$\frac{xy}{x+c}\Delta t$.

2- The expectation $E(\Delta x) = \sum_{i=1}^{4} p_i \Delta x_i$ is 2×1 matrix, the expectation can be expressed as follows.

$$E(\Delta x) = \sum_{i=1}^{4} p_i \Delta x_i = p_1 \Delta x_1 + p_2 \Delta x_2 + p_3 \Delta x_3 + p_4 \Delta x_4 \,,$$

$$E(\Delta x) = \sum_{i=1}^{4} p_i \Delta x_i = rx \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \left(\frac{rx^2}{K} + \frac{xy}{x+c}\right)\begin{pmatrix} -1 \\ 0 \end{pmatrix} + by \begin{pmatrix} 0 \\ -1 \end{pmatrix} + \left(\frac{xy}{x+c}\right)\begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

$$E(\Delta x) = \begin{pmatrix} rx \\ 0 \end{pmatrix} + \begin{pmatrix} -(\frac{rx^2}{K} + \frac{xy}{x+c}) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -by \end{pmatrix} + \begin{pmatrix} 0 \\ (\frac{xy}{x+c}) \end{pmatrix}.$$

$$E(\Delta x) = \begin{pmatrix} rx - (\frac{rx^2}{K} + \frac{xy}{x+c}) \\ -by + (\frac{xy}{x+c}) \end{pmatrix} \Delta t.$$

3- The covariance matrix, can be expressed as follows.

$$E(\Delta x(\Delta x)^T) = \sum_{i=1}^{4} p_i \Delta x_i (\Delta x_i)^T.$$

$$= p_1 \Delta x_1 (\Delta x_1)^T + p_2 \Delta x_2 (\Delta x_2)^T + p_3 \Delta x_3 (\Delta x_3)^T + p_4 \Delta x_4 (\Delta x_4)^T.$$

$$= rx \begin{pmatrix} 1 \\ 0 \end{pmatrix} (1 \;\; 0) + \left(\frac{rx^2}{K} + \frac{xy}{x+c}\right) \begin{pmatrix} -1 \\ 0 \end{pmatrix} (-1 \;\; 0) + by \begin{pmatrix} 0 \\ -1 \end{pmatrix} (0 \;\; -1) + \left(\frac{xy}{x+c}\right) \begin{pmatrix} 0 \\ 1 \end{pmatrix} (0 \;\; 1).$$

$$E(\Delta x(\Delta x)^T) = \begin{pmatrix} rx \\ 0 \end{pmatrix} (1 \;\; 0) + \begin{pmatrix} -\left(\frac{rx^2}{K} + \frac{xy}{x+c}\right) \\ 0 \end{pmatrix} (-1 \;\; 0) + \begin{pmatrix} 0 \\ -by \end{pmatrix} (0 \;\; -1) + \begin{pmatrix} 0 \\ \left(\frac{xy}{x+c}\right) \end{pmatrix} (0 \;\; 1).$$

$$E(\Delta x(\Delta x)^T) = \begin{pmatrix} rx & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} (\frac{rx^2}{K} + \frac{xy}{x+c}) & 0 \\ 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & by \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & (\frac{xy}{x+c}) \end{pmatrix}.$$

$$E(\Delta x(\Delta x)^T) = \begin{pmatrix} rx + (\frac{rx^2}{K} + \frac{xy}{x+c}) & 0 \\ 0 & by + (\frac{xy}{x+c}) \end{pmatrix} \Delta t.$$

$$E(\Delta x(\Delta x)^T) = \begin{pmatrix} rx + (\frac{rx^2}{K} + \frac{xy}{x+c}) & 0 \\ 0 & by + (\frac{xy}{x+c}) \end{pmatrix} \Delta t = V \Delta t.$$

4 – Formulate the stochastic system as .

$$dX(t) = f(X(t), t)dt + h(X(t), t)dW(t).$$

where;

$$dX(t) = \begin{bmatrix} dx_t \\ dy_t \end{bmatrix}, f(X(t), t) = \left[\frac{E(\Delta X)}{\Delta t}\right], h(X(t), t) = \sqrt{V} \text{ and } dW(t) = \begin{bmatrix} dW_1(t) \\ dW_2(t) \end{bmatrix}.$$

$$\begin{pmatrix} dx_t \\ dy_t \end{pmatrix} = \begin{pmatrix} rx_t - (\frac{rx_t^2}{K} + \frac{x_t y_t}{x+c}) \\ -by_t + (\frac{x_t y_t}{x_t + c}) \end{pmatrix} dt + \begin{pmatrix} \sqrt{rx + (\frac{rx^2}{K} + \frac{xy}{x+c})} & 0 \\ 0 & \sqrt{by + (\frac{xy}{x+c})} \end{pmatrix} \cdot \begin{pmatrix} dW_1(t) \\ dW_2(t) \end{pmatrix}.$$

$$\begin{cases} dx_t = \left(rx_t - (\frac{rx_t^2}{K} + \frac{x_t y_t}{x_t + c})\right) dt + \sqrt{rx + (\frac{rx^2}{K} + \frac{xy}{x+c})}\, dW_1(t). \\ dy_t = \left(-by_t + (\frac{x_t y_t}{x_t + c})\right) dt + \sqrt{by + (\frac{xy}{x+c})}\, dW_2(t) . \end{cases}$$

The numerical representation of the trajectories of this stochastic model is shown in Figure 2.

The equivalent system for the former system.

6- The diffusion matrix G of dimension 2×4 is:

$$G = \begin{pmatrix} \sqrt{rx} & -\sqrt{\left(\frac{rx^2}{K} + \frac{xy}{x+c}\right)} & 0 & 0 \\ 0 & 0 & -\sqrt{by} & \sqrt{\left(\frac{xy}{x+c}\right)} \end{pmatrix}.$$

$$dX(t) = f(X(t), t)dt + g(X(t), t)dW(t).$$

where;

$$dX(t) = \begin{bmatrix} dx_t \\ dy_t \end{bmatrix}, f(X(t),t) = \begin{bmatrix} \frac{E(\Delta X)}{\Delta t} \end{bmatrix}, g(X(t),t) = G \text{ and } dW(t) = \begin{bmatrix} dW_1(t) \\ dW_2(t) \\ dW_3(t) \\ dW_4(t) \end{bmatrix}.$$

Thus, the system takes the following form:

$$\begin{pmatrix} dx_t \\ dy_t \end{pmatrix} = \begin{pmatrix} rx_t - (\frac{rx_t^2}{K} + \frac{x_t y_t}{x+c}) \\ -by_t + (\frac{x_t y_t}{x_t+c}) \end{pmatrix} dt + \begin{pmatrix} \sqrt{rx} & -\sqrt{\left(\frac{rx^2}{K} + \frac{xy}{x+c}\right)} & 0 & 0 \\ 0 & 0 & -\sqrt{by} & \sqrt{\left(\frac{xy}{x+c}\right)} \end{pmatrix} \cdot \begin{pmatrix} dW_1(t) \\ dW_2(t) \\ dW_3(t) \\ dW_4(t) \end{pmatrix}.$$

$$\begin{cases} dx_t = \left(rx_t - (\frac{rx_t^2}{K} + \frac{x_t\, y_t}{x_t+c})\right) dt + \sqrt{rx}\, dW_1(t) - \sqrt{\left(\frac{rx^2}{K} + \frac{xy}{x+c}\right)}\, dW_2(t). \\ dy_t = \left(-by_t + (\frac{x_t\, y_t}{x_t+c})\right) dt - \sqrt{by}\, dW_3(t) + \sqrt{\left(\frac{xy}{x+c}\right)}\, dW_4(t). \end{cases}.$$

This illustration captures the intricate relationship between predator and prey, where variations in one's population have an impact on the other. This fluctuating behavior demonstrates how random elements contribute to ecological balance and result in erratic species interactions, Figure3.

Numerical Representation:

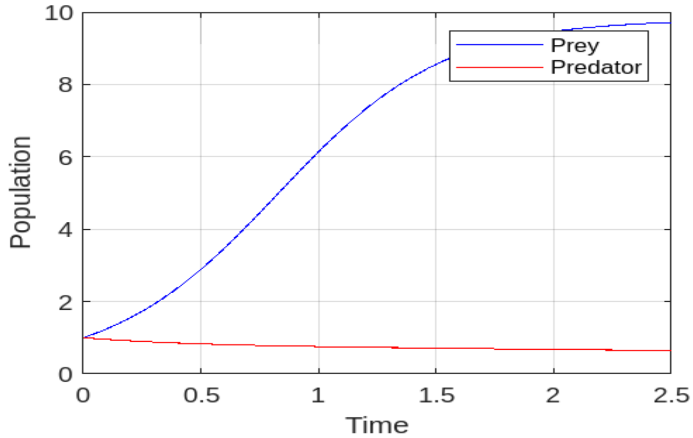

Figure 1. Prey-Predator in Deterministic Model *r*=3, b=1, c=1, k=10

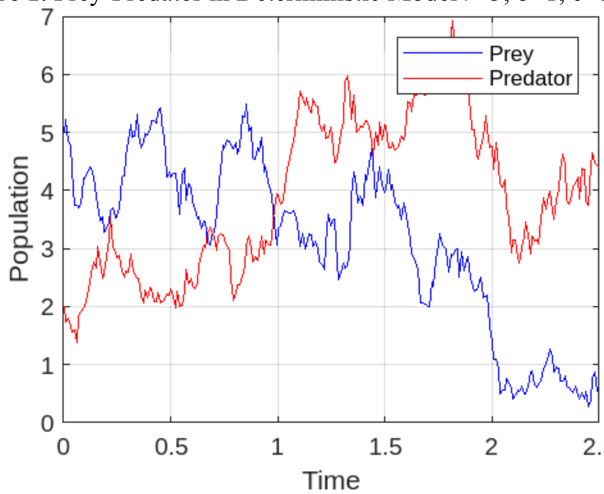

Figure 2. predation in Stochastic Model; *r*=1, b=1, c=1, k=10

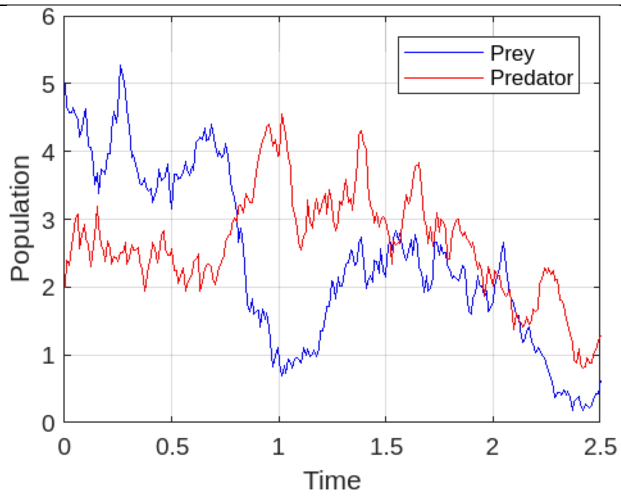

Figure 3. Predation in equivalent Stochastic Model; *r*=1, b=1, c=1, k=10

CONCLUSION

The aim of this study is to investigate the dynamic behaviors of a predator-prey system in which both the predator and the prey display herd characteristics. Deterministic environments are used to examine topics such as positivity, limitations, and equilibrium point stability. We normalized the prey species' birth rate and the predator species' death rate using Gaussian white noise in order to account for the impact of a changing environment. The correctness of our analytical results was confirmed by numerical simulations.

The predator-prey model has a significant impact on both our everyday lives and the ecosystem around us. It also helps us better understand fundamental ecological interactions and create practical solutions for protecting species from extinction and managing the environment sustainably.

CONFLICT OF INTEREST

We pronounce that we have no strife of intrigued.

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Appendix

1. Code of predator – prey (Deterministic model):

```
% Parameters
r = 3; % Intrinsic growth rate of prey
K = 10; % Carrying capacity of the environment for the prey
c = 1; % Constant related to the interaction term
b = 1; % Death rate of the predator in the absence of prey

% ODE system
pred_prey = @(t, Y) [
    r * Y(1) * (1 - Y(1)/K) - Y(1) * Y(2) / (Y(1) + c); % dx/dt
    Y(2) * (-b + Y(1) / (Y(1) + c))                    % dy/dt
];

% Initial conditions
x0 = 1; % Initial prey population
y0 = 1; % Initial predator population
Y0 = [x0; y0];

% Time span
tspan = [0 2.5];

% Solve ODE
[t, Y] = ode45(pred_prey, tspan, Y0);

% Plot results
figure;
plot(t, Y(:,1), '-b', 'DisplayName', 'Prey');
hold on;
plot(t, Y(:,2), '-r', 'DisplayName', 'Predator');
xlabel('Time');
ylabel('Population');
legend;
grid on;
hold off;
```

2. Code of predator – prey (Stochastic model):

```
% Parameters
r = 1; % Intrinsic growth rate of prey
K = 10; % Carrying capacity of the environment for the prey
c = 1; % Constant related to the interaction term
b = 1; % Death rate of the predator in the absence of prey

% Initial conditions
x0 = 5; % Initial prey population
y0 = 2; % Initial predator population
T = 2.5; % Total time
dt = 0.01; % Time step
N = T/dt; % Number of time steps
```

```
% Preallocate arrays
x = zeros(1, N);
y = zeros(1, N);
t = linspace(0, T, N);
x(1) = x0;
y(1) = y0;

% Simulation using Euler-Maruyama method
for i = 1:N-1
    % Deterministic part
    dx_det = (r * x(i) - (r * x(i)^2 / K + (x(i) * y(i)) / (x(i) + c))) * dt;
    dy_det = (-b * y(i) + (x(i) * y(i)) / (x(i) + c)) * dt;

    % Stochastic part
    dx_sto = sqrt(r * x(i) + (r * x(i)^2 / K + x(i) * y(i) / (x(i) + c))) * sqrt(dt) *
randn;
    dy_sto = sqrt(b * y(i) + (x(i) * y(i) / (x(i) + c))) * sqrt(dt) * randn;

    % Update populations
    x(i+1) = x(i) + dx_det + dx_sto;
    y(i+1) = y(i) + dy_det + dy_sto;

    % Ensure populations remain non-negative
    x(i+1) = max(x(i+1), 0);
    y(i+1) = max(y(i+1), 0);
end

% Plot results
figure;
plot(t, x, '-b', 'DisplayName', 'Prey');
hold on;
plot(t, y, '-r', 'DisplayName', 'Predator');
xlabel('Time');
ylabel('Population');
legend;
grid on;
hold off;
```

3. Code of predator – prey (equevelant Stochastic model):

```
% Parameters
r = 1; % Intrinsic growth rate of prey
K = 10; % Carrying capacity of the environment for the prey
c = 1; % Constant related to the interaction term
b = 1; % Death rate of the predator in the absence of prey

% Initial conditions
x0 = 5; % Initial prey population
y0 = 2; % Initial predator population
T = 2.5; % Total time
dt = 0.01; % Time step
N = T/dt; % Number of time steps

% Preallocate arrays
x = zeros(1, N);
y = zeros(1, N);
t = linspace(0, T, N);
x(1) = x0;
y(1) = y0;

% Simulation using Euler-Maruyama method
```

```
for i = 1:N-1
    % Deterministic part
    dx_det = (r * x(i) - (r * x(i)^2 / K + (x(i) * y(i)) / (x(i) + c))) * dt;
    dy_det = (-b * y(i) + (x(i) * y(i)) / (x(i) + c)) * dt;

    % Stochastic part
    dW1 = sqrt(dt) * randn;
    dW2 = sqrt(dt) * randn;
    dW3 = sqrt(dt) * randn;
    dW4 = sqrt(dt) * randn;

    dx_sto = sqrt(r * x(i)) * dW1 - sqrt((r * x(i)^2 / K + x(i) * y(i) / (x(i) + c))) *
dW2;
    dy_sto = -sqrt(b * y(i)) * dW3 + sqrt(x(i) * y(i) / (x(i) + c)) * dW4;

    % Update populations
    x(i+1) = x(i) + dx_det + dx_sto;
    y(i+1) = y(i) + dy_det + dy_sto;

    % Ensure populations remain non-negative
    x(i+1) = max(x(i+1), 0);
    y(i+1) = max(y(i+1), 0);
end

% Plot results
figure;
plot(t, x, '-b', 'DisplayName', 'Prey');
hold on;
plot(t, y, '-r', 'DisplayName', 'Predator');
xlabel('Time');
ylabel('Population');
legend;
grid on;
hold
```