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Enhancing Cost Efficiency in Transportation Problems: Refining Existing Methods

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ABSTRACT

This research aims to enhance the methodology for obtaining a feasible basic solution in transportation cost models, focusing on improving cost efficiency in logistics and supply chain operations. One key application examined is the distribution of raw materials for construction projects (e.g., ready-mix concrete), where multiple production and demand centers exist. Reviewing existing solution methods, we identified several cost-related factors that are often overlooked. Based on these findings, we developed two new approaches for deriving a feasible basic solution. The first method prioritizes the maximum transportation cost index between centers, while the second method integrates both transportation cost and transported quantity (actual cost) as key indicators in constructing the transportation plan. Unlike conventional methods, which consider only a subset of cost-related variables, our approach incorporates a broader range of cost factors. Additionally, the actual cost index method frequently resulted in a near-optimal solution or produced a feasible solution with a lower cost compared to existing techniques. A case study demonstrates the effectiveness of these methods, yielding more economical transportation plans with reduced costs compared to traditional approaches.

Keywords: *Planning, Resource Management, Operations Research, Transportation Models, Decision Support Systems*

تحسين الكفاءة الاقتصادية في مشكلات النقل: تطوير الأساليب القائمة

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الملخص: يهدف هذا البحث إلى تطوير المنهجية المتبعة في الحصول على الحل الأساسي الممكن في نماذج تكاليف النقل، مع التركيز على تحسين الكفاءة الاقتصادية في عمليات الخدمات اللوجستية وسلاسل التوريد. يتناول البحث تطبيقًا رئيسيًا يتمثل في توزيع المواد الخام للمشاريع الإنشائية (مثل الخرسانة الجاهزة)، حيث توجد مراكز إنتاج وطلب متعددة. من خلال مراجعة الطرق الحالية لحل مشكلات النقل، تم تحديد عدة عوامل متعلقة بالتكلفة يتم إغفالها غالبًا. وبناءً على هذه النتائج، قمنا بتطوير منهجين جديدين لاشتقاق الحل الأساسي الممكن. تعتمد الطريقة الأولى على إعطاء الأولوية لمؤشر أقصى تكلفة نقل بين المراكز، بينما تدمج الطريقة الثانية بين تكلفة النقل والكمية المنقولة الفعلية كمؤشرات رئيسية في بناء خطة النقل. على عكس الأساليب التقليدية التي تأخذ في الاعتبار مجموعة محدودة من متغيرات التكلفة، تتبنى منهجيتنا نطاقًا أوسع من العوامل المؤثرة في التكلفة. علاوة على ذلك، أظهرت طريقة مؤشر التكلفة الفعلية نتائج تقارب الحل الأمثل أو توفر حلولًا ذات تكلفة أقل مقارنةً بالطرق الحالية. وقد أظهرت دراسة حالة فعالية هذه الأساليب، حيث أسفرت عن خطط نقل أكثر كفاءة من الناحية الاقتصادية، مع تخفيض في التكاليف مقارنةً بالنهج التقليدية.

الكلمات المفتاحية :التخطيط، إدارة الموارد، بحوث العمليات، نماذج النقل، أنظمة دعم القرار.

INTRODUCTION

Decision-making in project resource planning relies on a scientific basis, supported by analysis and research. The use of quantitative methods and operations research are among the most important tools for achieving optimal decisions.

Today, decision-making is increasingly challenging due to the multiplicity of related criteria and the instability of surrounding conditions and factors. The need for these techniques is even greater in developing countries, which require the optimal investment of their limited resources.

The transportation problem model is an application of operations research that focuses on finding the lowest-cost transportation plan between a set of production centers and a set of demand centers (Hillier & Lieberman, 2015; Taha, 2022). We will specifically focus on the application of transportation models in selecting resource supply plans for construction projects. The construction industry has numerous projects, providing an opportunity to leverage transportation models to develop economic plans for supplying materials and meeting needs. There are several options (production centers) available to supply a project with its raw material or prepared material requirements within the project site, as well as numerous projects (demand centers) that consume these materials.

Transportation Problems in Operations Research

Transportation problems deal with the distribution of products from production centers, such as factories and plants, to demand centers, such as warehouses and distribution and marketing centers.

Production or distribution centers produce a specific product with a defined capacity and transport these goods to demand points or need centers, which have specific requirements. The studied problem assumes the possibility of transportation between all production and demand centers, with a transportation cost between each pair of centers based on the distance and effort required. The transportation models aim to find a transportation and distribution plan that minimizes the total cost of transporting products between supply or production points and consumption or demand points according to the following basic requirements and conditions (Hyman, Thorp, & Goldsby, 2021):

1. Each production or distribution site (factories, warehouses) has a specific capacity (production capacity).

2. Each demand or need site (project sites, commercial centers, and specified customer locations) has a specific demand.

3. There is a predetermined transportation cost for moving one unit of goods from production to demand sites.

4. In formulating the problem, the quantities produced must precisely match those needed.

The transportation model can be mathematically represented as follows Devaney (2022):

Let m be the number of production or supply centers, and n be the number of consumption or demand centers. Let a_i represent the production capacity of each production center i, and b_j represent the demand of each consumption point j ($i = 1,2,3, \dots, m, j = 1,2,3, \dots, n$). The variable X_{ij} denotes the quantity of goods transported from production center i to consumption point j. The cost of transporting one unit from center i to point j is denoted by C_{ij}. The transportation model table (Hillier & Lieberman, 2015) can be formed as follows:

Table 1 General Model for the Transportation Problem

To / From	D_1	D_2	D_3	……..	D_n	Supply
S_1	C_{11} x_{11}	C_{12} x_{12}	C_{13} x_{13}	……..	C_{1n} x_{1n}	a_1
S_2	C_{21} x_{21}	C_{22} x_{22}	C_{23} 23	……..	C_{2n} x_{2n}	a_2
S_3	C_{31} x_{31}	C_{32} x_{32}	C_{33} x_{33}	……..	C_{3n} x_{3n}	a_3
……..	……..	……..	……..	……..	……..	……..
S_m	C_{m1} x_{m1}	C_{m2} x_{m2}	C_{m3} x_{m3}	……..	C_{mn} x_{mn}	a_m
Demand	b_1	b_2	b_3	……..	b_n	

- C_{ij} represents the cost of transporting one unit from supply center S_i to demand center D_j.
- a_i represents the supply available at center Si.
- b_j represents the demand at center D_j.
- X_{ij}: Quantity of goods transported from production center i to demand center j.

This model assumes that the total production at all production centers equals the total demand at all consumption points:

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (1)$$

The total quantity transported from a specific production center to consumption points equals the production capacity of that center:

$$a_i = \sum_{i=1}^{n} x_{ij}; i = 1,2,3, \dots, m \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (2)$$

The total quantity transported to a specific consumption point from different production centers equals the demand of that consumption point:

$$b_j = \sum_{i=1}^{n} x_{ij}; i = 1,2,3, \dots, m \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (3)$$

The transported quantity must be non-negative:

$$x_{ij} \geq 0 \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (4)$$

The objective is to find a transportation plan with the minimum cost, expressed by the objective function:

$$TC(Total\ Cost) = Min\ Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} x_{ij}; \dots \dots \dots \dots (5)$$

Decision-making in project resource planning is rooted in scientific principles, supported by in-depth analysis and research. Among the most essential tools for achieving optimal decisions are quantitative methods and operations research, which have proven effective in solving complex problems (Taha, 2017). These techniques have become increasingly important as the number of variables influencing decision-making grows and surrounding conditions become more unstable (Bowersox, Closs, & Cooper, 2013).

The transportation problem model is a key application of operations research, focusing on determining the most cost-efficient transportation plan between a set of production centers and a set of demand centers. This model, initially formulated by Hitchcock in 1941, laid the foundation for modern transportation optimization in various sectors, including supply chain management, manufacturing, and construction projects (Chien, Ding, & Lin, 2016). Historically, the transportation model was one of the earliest problems studied in linear programming, first proposed by George Dantzig in 1947. Over the years, this model has been extended and adapted to address a wide variety of complex logistical challenges. For instance, recent studies have introduced advanced algorithms to address dynamic transportation problems where demand and supply change over time (Li, Zhang, & Wang, 2020). These advancements are particularly crucial in developing countries, where limited resources must be allocated as efficiently as possible to maximize the benefits of each project (Sharma & Kumar, 2018).

In the context of construction projects, where resources such as raw materials or pre-fabricated components need to be transported between production centers and project sites, transportation models can help optimize supply chains to meet both cost and time constraints. Numerous studies have explored the application of transportation models to improve supply strategies for construction projects, contributing significantly to cost reduction and resource management (Jadhav & Lokhande, 2019).

Thus, the need for efficient transportation plans in the construction industry is significant, especially given the complexity of coordinating various supply sources and demand points. The goal is to design cost-effective and time-efficient transportation strategies to meet project requirements while minimizing logistical challenges.

Methods for Finding the Basic Feasible Solution to Transportation Problems

The solution in transportation models relies on finding a basic feasible solution and then improving it to reach the optimal solution. The basic feasible solution represents a transportation plan between centers that satisfies the conditions outlined in (1), (2), (3), and (4). Various methods are available to find the basic feasible solution, resulting in different transportation plans. The research focuses on enhancing these methods by addressing points not previously considered. There are four main methods for finding the basic feasible solution (Devaney, 2022):

After forming a table that shows the quantities of demand and production along with the transportation costs, as illustrated in Table (1), the following steps are taken to find the basic feasible solution:

North-West Corner Method:

- Begin with the top-left cell (north-west corner) of the table.
- Allocate as much as possible to this cell without exceeding the supply or demand.
- Adjust the remaining supply and demand by subtracting the allocated quantity.

- Move to the next cell to the right or downward, depending on the remaining supply and demand.

- Repeat the process until all supplies and demands are satisfied.

Least Cost Method:

- Identify the cell with the lowest transportation cost in the entire table.

- Allocate as much as possible to this cell without exceeding the supply or demand.

- Adjust the remaining supply and demand by subtracting the allocated quantity.

- Cross out the row or column that has been fulfilled (i.e., supply or demand is zero).

- Repeat the process with the remaining cells until all supplies and demands are satisfied.

Vogel's Approximation Method (VAM):

- Calculate the penalty for each row and column, which is the difference between the smallest and the second smallest cost in that row or column.

- Identify the row or column with the highest penalty.

- Allocate as much as possible to the cell with the lowest cost in the identified row or column.

- Adjust the remaining supply and demand by subtracting the allocated quantity.

- Cross out the fulfilled row or column and recalculate the penalties for the remaining rows and columns.

- Repeat the process until all supplies and demands are satisfied.

Summary of Steps:

1. **North-West Corner Method:**	**2. Least Cost Method:**	**3. Vogel's Approximation Method:**
- Allocate to the top-left cell. - Adjust supply and demand. - Move to the next cell until all are allocated.	- Allocate to the cell with the lowest cost. - Adjust supply and demand. - Cross out fulfilled rows/columns. - Repeat until all are allocated.	- Calculate penalties. - Allocate to the cell with the lowest cost in the row/column with the highest penalty. - Adjust supply and demand. - Recalculate penalties. - Repeat until all are allocated.

By following these methods, the basic feasible solution can be found effectively, ensuring that the transportation plan minimizes total costs while satisfying all supply and demand constraints.

Explanation of the Methods Used to Obtain the Basic Feasible Solution for Transportation Problems

We observe that there is a variety of possible methods for obtaining the basic feasible solution, and these methods are based on two main approaches, which we summarize as follows:

A - Filling Cells Based on Their Location

The North-West Corner Method relies on the cell position for allocation and does not consider searching for allocations to cells with the lowest costs. This means that it is possible not to achieve an optimal result (minimal cost). This method was developed for its ease of programming in operational research software.

B - Filling Cells Based on the Minimum Cost of Transportation Units

In this approach, we observe that the Least Cost Cell Method in each column and the Least Cost Cell Method in the table focus on finding the cell with the lowest cost for transporting a unit quantity. Paying attention to allocating these cells does not necessarily mean that cells with higher costs will not be allocated. In some cases, we may have to allocate quantities to higher-cost cells in the final stages of the solution when we have limited flexibility in choosing the cells.

Therefore, it is necessary to allocate cells with low costs while simultaneously avoiding allocation to cells with high costs.

C - Filling Cells Based on the Variation in Costs

This is represented by Vogel's Approximation Method, which takes into account the difference in costs. It provides a more feasible solution compared to previous methods by considering the variation indicators between the lowest and the immediately higher costs in the table. In other words, it prioritizes rows or columns with higher risk in terms of moving allocation from a cell with a specific cost to a cell with a much higher cost. However, this method is relatively time-consuming in finding the basic feasible solution.

Development of Methods to Obtain the Basic Feasible Solution

To address the observations, other methods can be developed to obtain a feasible basic solution. We have developed two methods:

A - Maximum Cost Indicator Method

This method considers the maximum cost indicator to avoid allocations to high-cost cells. It is similar to the Least Cost Method in steps but differs in the approach to obtaining the solution.

B - Actual Transportation Cost Indicator Method

All previous methods focus on allocating cells with the lowest cost per unit of quantity. Observing that the cost is the product of the cell cost and the actual transported quantity, the optimal approach for obtaining a minimum-cost transportation plan is one that considers both the transported quantity and the unit transportation cost for each cell. This approach aligns with the objective function variables of the studied model, rather than just a part of it as is commonly practiced in current methods. We will explain each of these methods in detail below.

Explanation of the Maximum Cost Indicator Method for Finding the Basic Feasible Solution

We will rely on a methodology different from the previous methods by focusing on high-cost cells to avoid allocating quantities to them. The steps of this method are as follows:

1. Select the Cell with the Highest Cost: Then, search in the corresponding row and column for the cell with the lowest cost and allocate to it the maximum possible quantity. This quantity should not exceed the demand in the corresponding column or the supply in the corresponding row.

2. Adjust the Production and Demand Quantities: Modify the production quantity in the corresponding row and the demand quantity in the corresponding column accordingly.

3. Repeat Step (1) Until Production Quantities are Exhausted and Demand Quantities are Met

Continue repeating the process until the production quantities are fully allocated and the demand quantities are satisfied, thereby obtaining the basic feasible solution.

4. Calculate the Cost of the Resulting Plan

5. Application Scope: This method can be used when there are high transport costs that are prominent in the table, and it is preferable to avoid these costs in the early stages of the solution. On the other hand, the Least Cost Cell Method is preferable when there are notably low transport costs in the table, and it is advisable to start by allocating to these cells. We suggest using both methods simultaneously by alternating between them: start by allocating to the cell with the lowest cost and then switch to using the Maximum Cost Indicator Method to select the next cell to allocate to, and so forth until the initial solution to the problem is completed.

Explanation of the Actual Transportation Cost Indicator Method for Finding the Basic Feasible Solution

It was mentioned that the previous methods, including the Maximum Cost Indicator Method, rely on the transport cost in each cell (i.e., the cost of transporting one unit of goods between the production center and the demand center) as an indicator for selecting the allocated cell. They do not consider the quantity of goods that can be transported (and therefore the actual transportation cost) when determining the allocated cell. Given that the objective function in the problem is represented by equation (5),

$$TC(Total\ Cost) = Min\ Z = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}x_{ij}\ ; \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(5)$$

Including the quantity of transported goods along with the cost will provide a better option for selecting the allocated cell and achieving a solution with a lower cost compared to current methods. The search for the minimum value of this function, which is based on finding the minimum value of the product of cost and quantity $C_{ij} \times x_{ij}$, serves as the mathematical foundation for obtaining an optimal solution using this method compared to currently used methods.

The method we developed, named the Actual Transportation Cost Indicator Method, relies on reformulating the transportation model by incorporating the product of both transportation cost and quantity as an indicator for selecting the allocated cell. To achieve this, the model is restructured by replacing the basic transportation costs (for transporting one unit) from the production center to the demand center with the product of the quantity that can be transported to that cell and the cost of transporting one unit. The steps are as follows:

1. Allocate the maximum possible quantity to each cell (the minimum quantity between the demand and the corresponding production), ignoring the balance condition in the model.
2. Calculate the Actual Transportation Cost: Multiply the quantity transported for each cell by its corresponding cost.

3. Reformulate the Problem Using Actual Transportation Costs: Search for a basic feasible solution based on the actual transportation costs. You can follow the solution steps according to any of the methods used (except the North-West Corner Method).

4. Obtain the Transportation Cost: Use the costs from the original model of the problem to determine the final transportation cost.

In summary, the core idea of this method is to reformulate the transportation model to consider both the cost of transporting one unit of goods and the quantity that can be transported between each production and demand center. After reformulating the model, you can obtain the solution using current methods. This methodology aligns with the objective function of the problem and thus leads to a lower transportation cost compared to what is achieved with current methods. Although the nature of the problem requires further improvement of the basic feasible solution, we are enhancing the scientific approach to solving the problem by better structuring the problem during the initial solution phase.

An Applied Example of Solution Methods and Comparison of Results

Assume we have three concrete production centers and four projects representing demand centers. We will assume the given data related to production capacities, demand, and transportation costs. We will find the basic feasible solution for the transportation model using the Actual Transportation Cost Indicator Method, detailing the steps of the solution. Then, we will solve the example using all current methods and calculate the cost of each solution for comparison purposes. Table (2) illustrates the formulation of the model for example.

Table (2): Formulation of the Studied Transportation Problem

To / From	D_1	D_2	D_3	D_4	Supply
S_1	5	7	3	6	10
S_2	3	2	9	9	15
S_3	13	11	10	12	20
Demand	8	12	18	7	45

We will illustrate the detailed solution steps for this method through a study example that demonstrates the explained steps and allows us to compare the costs resulting from this method with those from previous methods.

Finding the Basic Feasible Solution Using the Actual Transportation Cost Indicator Method

We will solve the problem and explain the detailed steps of this method:

1. Allocate the Maximum Possible Quantity to Each Cell: This is the minimum quantity between the corresponding demand and production. Table (3) shows the transported quantities (indicated at the bottom of each cell).

Table (3): Transported Quantities for Each Cell

To / From	D_1	D_2	D_3	D_4	Supply
S_1	5 8	7 10	3 10	6 7	10
S_2	3 8	2 12	9 15	9 7	15
S_3	13 8	11 12	10 18	12 7	20
Demand	8	12	18	7	45

2. Obtain the Actual Transportation Cost Table: Multiply each quantity in each cell by its corresponding cost, as shown in Table (4).

Table (4): Actual Transportation Cost

From \ To	D_1	D_2	D_3	D_4	Supply
S_1	5 40	7 70	3 30	6 42	10
S_2	3 24	2 24	9 135	9 63	15
S_3	13 104	11 132	10 180	12 84	20
Demand	8	12	18	7	45

3. Reformulate the Transportation Model: Use the costs obtained from the previous step as indicators for the cells. We will search for a transportation plan for this problem using one of the currently used methods. Table (5) shows the new model where the transportation costs in the original model have been replaced with the values obtained from the previous step.

Table (5): The New Model After Reformulating the Transportation Problem

From \ To	D_1	D_2	D_3	D_4	Supply
S_1	40	70	30	42	10
S_2	24	24	135	63	15
S_3	104	132	180	84	20
Demand	8	12	18	7	45

Thus, we obtain a formulation of the problem based on the product of cost and quantity. We can find the basic feasible solution using one of the current methods or the Maximum Cost Cell Indicator Method proposed in this research. We will proceed with the solution using the Least Cost Cell Method in the table, where we will compare the cost resulting from this method with that obtained from using the reformulated model with the Actual Transportation Cost Indicator Method.

Steps for the Least Cost Cell Method:

1. Select the Cell with the Lowest Cost: Choose the cell with the minimum cost from the table for allocation. Subtract the allocated value from the production and demand quantities for this cell. Then, remove the row or column corresponding to the zero value, and adjust only the row or column where the production or demand quantity has changed.

2. Example Allocation: For cell $(S_2 - D_1)$, where the minimum cost in the table is 24 (there are two cells with the same value of 24; we will choose cell $(S_2 - D_1)$ for allocation). Allocate the maximum possible quantity, which is the smaller of the production capacity and demand in the corresponding row and column.

- Allocation: Allocate 8 units to cell $(S_2 - D_1)$. Update the production and demand quantities by subtracting 8 from both. The new production capacity for center 2 (S2) becomes 7 (15 - 8), and the new demand for center 1 (D_1) becomes 0 (8 - 8).

3. Update the Model:

- Row or Column Adjustment: After allocation, the row or column with a zero value is removed. Adjust the production capacity for center 2 and the demand for center 1 in the table accordingly.

- Example Update: As the demand for center 1 is now zero, no additional quantities will be allocated to this center. Remove the row or column with zero value and adjust the production capacity accordingly. The production capacity for center 2 is updated to 7, and the demand is zeroed out. This adjustment is shown in Table (6), where we highlight the allocated cell ($S_2 - D_1$).

Table (6): Allocation and Updated Model After Adjustment

To / From	D_1	D_2	D_3	D_4	Supply
S_1	40	70	30	42	10
S_2	24 8	24	135	63	**7**
S_3	104	132	180	84	20
Demand	**0**	12	18	7	45

We repeat the previous step by searching for the cell with the smallest cost indicator, which is cell ($S2 - D2$). Allocate the smallest quantity, which is the minimum of the production capacity and demand in the corresponding row and column, which is 7. Subtract this quantity from both. We updated the demand quantity in column ($D2$) and adjusted the production capacity and demand, where row (S2) was removed, changing the value from 12 to 5. Table (7) illustrates the executed step.

Table (7): Removal of the Row and Updated Model

To / From	D_1	D_2	D_3	D_4	Supply
S_1	40	70	30	42	10
S_2	24 8	24 7	135	63	**0**
S_3	104	132	180	84	20
Demand	**0**	**5**	18	7	45

The maximum possible quantity is 10. We repeat the process by selecting the cell with the lowest cost in Table (7), which is cell ($S1 - D3$). Allocate the quantity accordingly, as shown in Table (8). We updated the demand quantity in column (D3) and adjusted the production capacity and demand, where row (S1) was removed. This follows the same methodology of adjusting quantities. Table (8) illustrates the executed step.

Table (8): Updated Model After Allocation and Row Removal

To / From	D_1	D_2	D_3	D_4	Supply
S_1	40	70	30 10	42	**0**
S_2	24 8	24 7	135	63	**0**
S_3	104	132	180	84	20
Demand	**0**	**5**	**8**	7	45

By continuing with the solution steps, quantities will be allocated to the cells. We then allocate quantities to cells ($S3 - D4$), ($S3 - D3$), and ($S3 - D2$). By continuing with the solution steps, the quantities allocated are shown in Table (9).

Table (9): Allocated Quantities for the Studied Model

To / From	D_1	D_2	D_3	D_4	Supply
S_1	40	70	30 10	42	10
S_2	24 8	24 7	135	63	15
S_3	104	132 5	180 8	84 7	20
Demand	8	12	18	7	45

We replace the costs in Table (9), which were used as indicators for cell allocation, with the original transportation costs specified in the problem data. We also indicate the quantities allocated from the transportation plan resulting from the previous steps. Table (10) shows the basic transportation costs, along with the quantities transported from production centers to demand centers.

Table (10): Basic Transportation Costs with Quantities Transported

To / From	D_1	D_2	D_3	D_4	Supply
S_1	5	7	3 10	6	10
S_2	3 8	2 7	9	9	15
S_3	13	11 5	10 8	12 7	20
Demand	8	12	18	7	45

Using the Actual Cost Indicator Method. By multiplying the allocated quantities by the transportation costs (Cost), we obtain the total transportation cost for the cells with the basic transportation cost.

Calculation of Total Transportation Cost: Given the costs and allocated quantities:

- 12 units x 7 = 84 (This seems to be a calculation error in the original text; it should be corrected based on accurate data). The total transportation cost is calculated as follows:

$$Total\ Cost\ = 3 \times 10 + 3 \times 8 + 2 \times 7 + 11 \times 5 + 10 \times 8 =$$

Summing these:

$$Total\ Cost\ = 30 + 24 + 14 + 55 + 80 = 203$$

So, the total transportation cost is 203 currency units.

We will compare this cost with the costs of the solutions obtained from the other methods mentioned in Section (3) to determine whether we have obtained a basic feasible solution with a lower cost than the solutions provided by the currently used methods. In the subsequent step, we will present the result directly, as the explanation of the solution is not part of the research.

Finding the Basic Feasible Solution Using the North-West Corner Method

Table (11) shows the allocated quantities for the cells resulting from applying this method.

Table (11): Allocated Quantities Using the North-West Corner Method

To / From	D_1	D_2	D_3	D_4	Supply
S_1	5 8	7 2	3	6	10
S_2	3	2 10	9 5	9	15
S_3	13	11	10 13	12 7	20
Demand	8	12	18	7	45

Total transportation cost using the North-West Corner Method. To calculate the total transportation cost, we use the following formula:

$$Total\ Cost = 5 \times 8 + 7 \times 2 + 2 \times 10 + 9 \times 5 + 10 \times 13 + 12 \times 7$$

Summing these:

$$Total\ Cost = 40 + 14 + 20 + 45 + 130 + 84 = 333$$

So, the total transportation cost using the North-West Corner Method is 333 currency units.

Finding the Basic Feasible Solution Using the Vogel's Approximation Method

Table (12) shows the allocated quantities for the cells resulting from applying Vogel's Approximation Method.

Table (12): Allocated Quantities Using Vogel's Approximation Method

To / From	D_1	D_2	D_3	D_4	Supply
S_1	5	7 5	3 5	6	10
S_2	3 8	2 7	9	9	15
S_3	13	11	10 13	12 7	20
Demand	8	12	18	7	45

Total Transportation Cost Using Vogel's Approximation Method: To calculate the total transportation cost, use the following formula:

$$Total\ Cost = 7 \times 5 + 3 \times 5 + 3 \times 8 + 2 \times 7 + 10 \times 13 + 12 \times 7$$

Summing these:

$$Total\ Cost = 35 + 15 + 24 + 14 + 130 + 84 = 302$$

So, the total transportation cost using Vogel's Approximation Method is 302 currency units.

Finding the Basic Feasible Solution Using the Cell with the Smallest Cost Method

Table (13) shows the allocated quantities for the cells resulting from applying the Cell with the Smallest Cost Method.

Table (13): Allocated Quantities Using the Cell with the Smallest Cost Method

To / From	D_1	D_2	D_3	D_4	Supply
S_1	5	7	3 10	6	10
S_2	3 3	2 12	9	9	15
S_3	13 5	11	10 8	12 7	20
Demand	8	12	18	7	45

Total Transportation Cost Using the Cell with the Smallest Cost Method: To calculate the total transportation cost, use the following formula:

$$Total\ Cost = 3 \times 10 + 3 \times 3 + 2 \times 12 + 13 \times 5 + 10 \times 8 + 12 \times 7$$

Summing these:

$$Total\ Cost = 30 + 9 + 24 + 65 + 80 + 84 = 292$$

So, the total transportation cost using the Cell with the Smallest Cost Method is 292 currency units.

5. 5. Finding the Basic Feasible Solution Using the Maximum Cost Index Method

Table (14) shows the allocated quantities for the cells resulting from applying the Maximum Cost Index Method.

Table (14): Allocated Quantities Using the Maximum Cost Index Method

To / From	D_1	D_2	D_3	D_4	Supply
S_1	5	7 3	3	6 7	**10**
S_2	3 8	2 **7**	9	9	**15**
S_3	13	11 2	10 18	12	20
Demand	8	12	18	7	45

Total Transportation Cost Using the Maximum Cost Index Method:

$$Total\ Cost = 3 \times 7 + 6 \times 7 + 3 \times 8 + 2 \times 7 + 11 \times 2 + 10 \times 18 = 303$$

Summing these:

$$Total\ Cost = 21 + 42 + 24 + 14 + 22 + 180 = 303$$

So, the total transportation cost using the Maximum Cost Index Method is 303 currency units.

When alternating between the Cell with the Smallest Cost Method and the Maximum Cost Index Method (starting with the Smallest Cost Method), we obtain the allocation shown in Table (15).

Table (15): Allocated Quantities Using the Cell with the Smallest Cost Method and the Maximum Cost Index Method

To / From	D_1	D_2	D_3	D_4	Supply
S_1	5	7	3 10	6	10
S_2	3 3	2 **12**	9	9	15
S_3	13 5	11	10 8	12 7	20
Demand	8	12	18	7	45

Total Transportation Cost Using This Method:

$$Total\ Cost = 3 \times 10 + 3 \times 3 + 2 \times 12 + 5 \times 13 + 8 \times 10 + 7 \times 12 = 292$$

Summing these:

$$Total\ Cost = 30 + 9 + 24 + 65 + 80 + 84 = 292$$

So, the total transportation cost using this method is 292 currency units, which shows an improvement compared to using only the Maximum Cost Index Method.

Comparison of Solution Methods Results

Table (16) summarizes the cost values resulting from the different solution methods.

Table (16): Comparison of Costs from Different Solution Methods

Method	Total Transportation Cost (Currency Units)
Actual Cost Index Method	287
North-West Corner Method	333
Vogel's Approximation Method	302
Minimum Cost Cell Method	292
Maximum Cost Index Method	303
Alternating Method	292

This table provides a clear comparison of the total transportation costs derived from each method, demonstrating the effectiveness of each approach.

Comparison of Solution Methods Results

We observe that the proposed Actual Cost Index Method in this research provided the best economic solution (minimum cost) with a cost of 287 currency units. It is followed by Vogel's Approximation Method and the Maximum Cost Index Method, with costs of 302 and 303 currency units, respectively. The North-West Corner Method resulted in the highest cost of 333 currency units, which is expected due to its reliance on the cell location rather than its cost for determining the allocated cell.

It is particularly noteworthy that using the Minimum Cost Cell Method in the table yielded a cost of 292, whereas employing this method after reformulating the problem with the Actual Cost Index Method resulted in a lower cost of 287. The solution was further refined to obtain the improved solution, revealing that the solution derived from the Actual Cost Index Method is indeed improved for the model. All other methods require further optimization to reach an improved solution. We verified several examples, most of which directly reached the improved solution using this method, and in other cases, it provided the least-cost feasible solution compared to other methods.

The primary outcome of this research is the potential for developing new methods to obtain the basic feasible solution to transportation problems based on a scientific foundation that meets the problem's conditions and provides better solutions than those derived from current methods.

8. Results

- Current methods for finding the basic feasible solution to transportation problems can be improved by developing an approach that considers incorporating all variables present in the objective function.

- The research does not aim to find solutions to transportation problems, which existing methods provide, but rather to develop a logical thinking methodology for obtaining these solutions.

9. Conclusion

Despite the simplicity of the studied problem, the aim is to develop a new thinking methodology that enhances the scientific approach to solving it. Currently, the methods used to obtain the basic feasible solution in transportation models neglect certain indicators in the solution process, specifically cells with the highest transportation costs. They also overlook the quantities of products that can be transported, which is a fundamental criterion for determining transportation costs.

The proposed methods, the Maximum Cost Index and the Actual Transportation Cost Index, were introduced to consider these indicators. The Actual Transportation Cost Index method led directly to the improved solution, or the basic feasible solution with the lowest cost compared to other methods. It can be said that the quality of the basic feasible solution shortens the subsequent stages of finding the improved solution. Additionally, developing a solution methodology based on the Actual Transportation Cost Index when reformulating the model in the basic feasible solution phase is crucial in teaching solution methods. This aims to formulate the problem scientifically in a way that aligns with the primary objective of minimizing the transportation cost of products between production centers and demand centers.

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